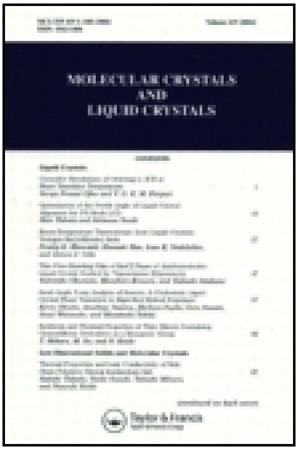

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Stability of One-Dimensional Director Structures in Ferromagnets and Liquid Crystals

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Stability of One-Dimensional Director Structures in Ferromagnets and Liquid Crystals

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Abstract—Director structures in an infinite slab in the presence of a magnetic field are considered. The director is pinned parallel to the slab faces on one of the faces and is free on the other face. It is shown that when the field is parallel to the direction of pinning, the only stable state is that of a uniform director field. It follows that in ferromagnetic slabs pinned by exchange anisotropy on one side, it is impossible to introduce a multiturn helix by rotation in a uniform magnetic field. A similar result follows for a nematic liquid crystal pinned by the Chatelain method.

The stability of one-dimensional ferromagnetic structures has been considered by Brown and Shtrikman.[1] They showed that under strict one-dimensional conditions (possible only in a slab of infinite traverse dimensions), all solutions in which the magnetization **M** is nonuniform are unstable. The purpose of the present paper is to augment the study of Brown and Shtrikman by relaxing one of the restrictions assumed in their treatment: namely, we allow, on one

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of the faces of our slab, a surface anisotropy so strong that it pins the magnetization to a particular direction parallel to the surface; we take this direction as the z axis. Bulk anisotropy is allowed, provided an easy axis is along z. The magnetic field is uniform and parallel to z. Under these conditions the state of uniform magnetization along z is a state of equilibrium[2] and is in fact the state of lowest free energy; for in this state the essentially nonnegative exchange energy and magnetostatic self-energy are zero, and the crystalline anisotropy energy and the energy in the external field are minimized.

Now consider any other state $\mathbf{M}_0(x)$. We have because of the pinning at $x = 0$

$$M_x(0) = M_y(0) = 0. \tag{1}$$

To test whether this state can be one of stable equilibrium, we must consider a slightly varied state $\mathbf{M}_\delta$ described by continuous functions (with piecewise-continuous derivatives) and still satisfying the boundary condition (1). For the state $\mathbf{M}_\delta$ we take

$$\begin{aligned} M_x(x) = M_y(x) &= 0, \qquad 0 \leqslant x \leqslant \delta, \\ \mathbf{M}(x) = \mathbf{M}_0(x-\delta)&, \qquad \delta \leqslant x \leqslant d, \end{aligned} \tag{2}$$

where d is the thickness of the slab. Since the distribution is one-dimensional, the magnetostatic self-energy can be treated as localized anisotropy energy.[1] Then the energy of the region $\delta \leqslant x \leqslant d$ in state $\mathbf{M}_\delta$ is the same as the energy of the region $0 \leqslant x \leqslant d-\delta$ in state $\mathbf{M}_0$; and the energy of the region $0 \leqslant x \leqslant \delta$ in state $\mathbf{M}_\delta$ is less than or at most equal to the energy of the region $d-\delta \leqslant x \leqslant d$ in state $\mathbf{M}_0$. Thus if $\mathbf{M}_0$ is a state of equilibrium, its equilibrium is unstable or at best neutral (in the latter case, successive variations δ of the same type will ultimately produce instability, provided $\mathbf{M}_\delta$ differs anywhere from $\mathbf{M}_0$).

Thus we have shown that the only y- and z-independent stable equilibrium, for our (centrosymmetric) ferromagnetic slab pinned on one side, is the uniform state.

What are the physical implications of our proof? Consider a ferromagnetic slab exchange-coupled[3] to an antiferromagnetic one in such a way that the magnetization on the ferro–antiferro interface is parallel to the interface. What will happen if we rotate this composite through several revolutions about an axis perpendicular to its surface, in a very strong uniform magnetic field (small, however, compared with the antiferromagnetic spin-flop field) parallel to the surface? Heuristically or intuitively, one may at first think that a helix is squeezed in, whose total angle of rotation is equal to the total angle of the field rotation. Our proof shows that this cannot be the case (at least if the contemplated final state is of the simple type in which **M** is independent of y and z), and that after a full rotation, we must somehow return to the original state. Note that the same conclusion applies if the rotation axis is, say, in the plane of the slab and perpendicular to the magnetic field, or if the field rather than the slab is rotated.

The results may be applied also to liquid crystals.[4] The hydrodynamic liquid-crystal equations are given in terms of the director **n**, which is a unit vector parallel to the local direction of molecular alignment. In terms of the director, the free energy of the " isotropic " nematics[5] is given, in the present problem,[6] by the same expression as the exchange energy in an isotropic ferromagnet. Thus we can take over what was done in the ferromagnet to the nematic liquid crystal. Pinning in this case is achieved by the Chatelain method, namely placing the liquid crystal on a glass plate which has been rubbed in one direction.[7] We can again use a rotating magnetic field to start twisting the molecules, as in many cases,[8] the anisotropy of the diamagnetic long molecules tends to make their long axes parallel to the field. Because of the tensor character of the interaction, we must in this case return to the original state after a rotation by π. Again we cannot produce a helix (at least of the y- and z-independent type) by rotating a uniform field.

If the magnetization or the director is pinned on both faces (with the same pinning direction on both), the instability theorem again holds for states $\mathbf{M}_0$ such that $\mathbf{M}_0(d-x) = \mathbf{M}_0(x)$. In this case, we consider a varied state M_δ described by Eqs. (2) for $0 \leqslant x \leqslant d/2$ and by $\mathbf{M}_\delta(d-x) = \mathbf{M}_\delta(x)$ for $d/2 \leqslant x \leqslant d$. Thus rotation of such a plate in a field, or rotation of the field, cannot produce a helix at least of the simple type one might intuitively expect.

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2. Here, as in Ref. 1, it is assumed that our ferromagnet has a center of symmetry. If this is not the case, the antisymmetric exchange will cause the appearance of an energy term that may result in a nonuniform equilibrium. This is closely related to the cholesteric liquid crystals; see Goshen, Mukamel, and Shtrikman, *Sol. State Comm.* **9**, 649 (1971).
3. For a review of exchange anisotropy see, for example, Jacobs, I. S. and Bean, C. P., in *Magnetism*, G. T. Rado and H. Suhl, ed., Academic Press, New York, 1963, Vol. 3, p. 271.
4. For a review see, for example, Chistyakov, I. G., *Sov. Phys. Usp.* **9**, 551 (1967).
5. de Gennes, P. G., in *Proc. Second Internatl. Liquid Crystals Conf., Kent State Univ., 1968*, Gordon and Breach, 1969; published also in *Molecular Crystals* **7**, 325–345 (1969). In the " isotropic " case $K_{11} = K_{22} = K_{33}$.
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